Search for correlation effects in linear chains of trapped ions

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We report a precise search for correlation effects in linear chains of 2 and 3 trapped Ca^+ ions. Unexplained correlations in photon emission times within a linear chain of trapped ions have been reported, which, if genuine, cast doubt on the potential of an ion trap to realize quantum information processing. We observe quantum jumps from the metastable $3d^2D_{5/2}$ level for several hours, searching for correlations between the decay times of the different ions. We find no evidence for correlations: the number of quantum jumps with separations of less than 10 ms is consistent with statistics to within errors of 0.05%; the lifetime of the metastable level derived from the data is consistent with that derived from independent single-ion data at the level of the experimental errors (1%); and no rank correlations between the decay times were found with sensitivity to rank correlation coefficients at the level of $|R| = 0.024$.

42.50.Lc, 42.50.Fx, 32.80.Pj, 32.70.Cs

The drive to realise the potential of quantum information processing [1,2] has led to the investigation of various experimental systems; among these is the ion trap, which has several advantages including the capability to generate entanglement actively with existing technology [3]. Following the proposal of an ion-trap quantum processor by Cirac and Zoller [4], several groups have carried out pioneering experiments [5–9]. In a recent review [10], the view was expressed that "the ion trap proposal for realizing a practical quantum computer offers the best chance of long term success." One of the attractive features of the trap is that the various interactions and processes which govern its behaviour have been exhaustively studied and are in principle well-understood. However, 14 years ago unexplained collective behaviour when several ions were present was reported [11]. This prompted tests in another laboratory which gave null results [12,13], but recently a further account of such effects has appeared [14]. There is thus an apparent conflict of evidence from different laboratories.

The effects manifest themselves as an enhanced rate of coincident quantum jumps. Sauter *et al.* [11] measured two- and three-fold coincident quantum jumps in a system of three trapped Ba^+ ions to occur two orders of magnitude more frequently than expected on the basis of statistics. This observation led to proposals that the ions were undergoing a collective interaction with the light field [11,15]. Itano *et al.* [12,13] subsequently made a search for such effects in groups of two and three Hg^+ ions in their laboratory. Their results were consistent with no correlations. In a test on two ions, when over 5649 consecutive jumps were observed, the number of apparent double jumps was 11, which was approximately the number that would be expected due to random coincidences within the finite time resolution of the experiment. Further tests based on photon statistics were also consistent with no correlations.

More recently, Block *et al.* [14] have observed an enhanced rate of two- and three-fold coincidences in a linear chain of ten Ca^+ ions, where the coincidences were not confined to adjacent ions. This led them to suggest an unexplained long range interaction between ions in the linear crystal. They also found that measurements of the lifetime τ of the $3D_{5/2}$ level (shelved state) from the 10-ion string produced discrepancies of as much as 6σ between runs under nominally identical conditions, where σ is the standard deviation for each run.

Since only the electromagnetic interaction is involved, it is extremely unlikely that these observations indicate new physics; nevertheless, they raise serious doubt about the suitability of the ion trap as a quantum information processing device. The coupling between a quantum system and its environment plays a crucial role in quantum information processing. An unexplained contribution to this coupling is especially significant, because any method to suppress the decoherence, such as quantum error correction (QEC) [2], relies on accurate knowledge of the process in question. It is furthermore particularly important to understand collective decoherence processes and place a reliable upper bound on their size, because the simultaneous combination of uncorrelated and correlated errors in a quantum computer poses the most severe constraints on QEC [16]. Thus, experimental reports of a decoherence process which is both unexplained and collective merit serious attention. We have therefore undertaken a search for the reported effects in linear chains of 2 and 3 trapped Ca^+ ions.

Our data were taken under conditions such that correlation effects would be expected on the basis of the results of [11] and [14], and are significantly more precise than either. We find no evidence at all for correlations.

Our work is complementary to that of [12,13] in that we are operating in a different system (Ca^+ instead of Hg^+) with a significantly different time-scale (mean rate for observed double quantum jumps of order 0.2 per minute instead of 2 per minute), and we perform several new statistical tests on 2 and 3 ions. Our upper bound for unexpected double jumps is 1.4 per hour, or 0.05% of the single jump rate. The corresponding upper bounds for the Hg^+ ion trap in [13] are 30 per hour and 0.06%.

The experimental method is very similar to that reported in our measurement of the lifetime of the $3d^2D_{5/2}$ level [17], which was originally adopted by Block *et al.* [14]. Linear crystals of a small number, N, of $^{40}Ca^+$ ions separated by about 15 μm are obtained by trapping in a linear Paul trap *in vacuo* ($\leq 2 \times 10^{-11}$ Torr), and laser-cooling the ions to a few mK. The transitions of interest are shown in figure 1. Laser beams at 397 nm and 866 nm continuously illuminate the ions, and the fluorescence at 397 nm is detected by a photomultiplier. The photon count signal is accumulated for bins of duration $t_b = 10.01$ ms (of which the last 2.002 ms is dead time), and logged. A laser at 850 nm drives the $3D_{3/2} - 4P_{3/2}$ transition. The most probable decay route from $4P_{3/2}$ is to the $4S_{1/2}$ ground state; alternatively, an ion can return to $3D_{3/2}$. However, about 1 decay in 18 occurs to $3D_{5/2}$, the metastable "shelving" level. At this point the fluorescence from the ion that has been shelved disappears. A shutter on the 850 nm laser beam remains open for 100 ms before it is closed, which gives ample time for shelving of all N ions. Between 5 and 10 ms after the shutter is closed we start to record the photomultiplier count signal in the 10 ms bins. We keep observing the photon count until it abruptly increases to a level above a threshold. This is set between the levels observed when 1 and 0 ions remain shelved. The signature for all N ions having decayed is taken to be ten consecutive bins above this threshold. After this we re-open the shutter on the 850 nm laser. This process is repeated for several hours, which constitutes one run.

The data from a given run were analysed as follows. The raw data consists of counts indicating the average fluorescence level in each bin of duration t_b (see figure 2). N thresholds λ_m are set, the m^{th} threshold being set between the levels observed when m and $(m-1)$ ions remain shelved. The number of bins observed below λ_N gives the decay time, t_N, of the first of N shelved ions to decay. The number of bins observed between λ_{m+1} and λ_m being exceeded gives the decay time, t_m, of the next ion to decay leaving $(m-1)$ ions shelved. The large number of t_m obtained are then gathered into separate histograms and the expected exponential distribution $A\exp(-\gamma_m t)$ is fitted to each, in order to derive the decay rate γ_m of the next ion to decay leaving $(m-1)$ ions shelved (see figure 3). It is appropriate to use a Poissonian fitting method (described in [17]), rather than least-squares, because of the small numbers involved in part of the distribution (at large t).

If the N ions are acting independently, each one will have a decay rate $\gamma = 1/\tau$, where τ is the lifetime of the $3D_{5/2}$ state. Since we do not distinguish between the fluorescence signals from the different ions, then with m ions remaining shelved the next decay is characterised by the increased rate $\gamma_m = m/\tau$.

Figure 3 shows the histogram of the decay times, t_1, of the second ion of two to decay obtained from a 3.2 hour run. The expected exponential decay fits the data very well. Events in the first bin of the histogram correspond to both ions being detected as decaying in the same bin, $t_1 = 0$. These quantum jumps, coincident within our time resolution, certainly do not occur two orders of magnitude more frequently than expected by random coincidence as was observed by Sauter *et al.* [11]. In fact, they are observed to occur less frequently than predicted by the fitted exponential to the histogram data. However, this is an artefact of our finite time resolution. The fitted exponential to the histogram data has value f_1 in the first bin, which gives the number of second ion decays that are expected to occur within t_b of the first ion decaying by random coincidence. However, for both ions to decay within a single bin, the second ion has an average time of less than t_b in which to decay. The exact details depend upon the analysis thresholds, λ_m, and the detector dead time. In the 2-ion case, one can show that, to first order in t_b/τ, the first bin width is modified to Ft_b where:

$$F = 0.98 - 0.8\lambda'_1 + 0.16{\lambda'_1}^2 + 0.16{\lambda'_2}^2 + 1.44\lambda'_2 - 0.64\lambda'_1\lambda'_2$$

with normalized thresholds:

$$\lambda'_m = \frac{\lambda_m - S_N}{S_{N-1} - S_N}$$

where S_m is the mean photon count with m ions shelved (so S_N is the mean background count level). This expression was verified using real and simulated data. The expected number of coincidences is therefore Ff_1. For the histogram shown, the 2-ion data was analyzed with the thresholds $\lambda'_1 = 1.4$ and $\lambda'_2 = 0.40$ (these are chosen to optimize the discrimination of the fluorescence levels S_m), which gives $F = 0.42$. The expected number of coincidences is $Ff_1 = 24 \pm 5$, assuming $\sqrt{n}$ errors, which agrees with the observed number of coincidences, 26. The second bin of the histogram is the only other bin expected to have a modified width, which is by a negligible amount. Note that, to ensure the number of coincidences is properly normalized, it is important that only events where at least $(m+1)$ ions were shelved at the start of an observation are included in the t_m histogram (for $m \neq N$).

Table I shows that the observed number of 2-fold coincidences in the 2- and 3-ion data agree with the expected value within $\sqrt{n}$ errors. The total expected number of 2-fold coincidences in all the data was 66.3 out

of 16132 quantum jumps observed to start with at least 2 ions shelved. We are therefore sensitive to changes in the proportion of 2-fold coincidences at the level of $\sqrt{66}/16132 = 0.05\%$ or about 1.4 event per hour.

The expected number of 3-fold coincidences depends on the threshold settings in a more complex way than in the 2-fold case, and here we simply use simulated 3-ion data to provide the predicted number of 3-fold coincidences shown in table I. The total number of expected 3-fold coincidences is 0.05 in both 3-ion data runs, which have a combined duration of 2.8 hrs. In fact, this predicted value is significantly lower than effects in our trap which can perturb the system sufficiently to cause de-shelving (such as collisions with residual background gas), as discussed in [17]. We observe at most one event, depending on the exact choice of threshold settings, and this does not constitute evidence for correlation.

The decay rates obtained from the 2- and 3-ion data are shown in figure 4, where the horizontal lines are the expected rates $\gamma_m = m/\tau$ assuming the ions to act independently. Combining all the γ_m derived from the 2- and 3-ion data as estimates of m/τ yields a value $\tau = 1177 \pm 10$ ms, where we include a 2 ms allowance for systematic error [17]. This is consistent with the value derived from single-ion data, $\tau = 1168 \pm 7$ ms [17]. We are therefore sensitive to changes in the apparent value of τ due to multiple ion effects at the level of 1%. Superfluorescence and subfluorescence as observed in a two-ion crystal [18] are calculated to be negligible with the large interionic distance of about 15 μm in the chain.

In order to look for more general forms of correlation between the decay times of each ion, rank correlation tests were performed. Table II gives the results; they show no significant correlations. The 2-ion data is the most sensitive, allowing underlying rank-correlation coefficients to be ruled out at the level of $|R_{12}| = 0.024$.

In summary, we have presented results that are consistent with no correlations of spontaneous decay within linear chains of 2 and 3 trapped Ca^+ ions, contrary to previous studies. First, the number of coincident quantum jumps were found to be consistent with those expected from random coincidence at the level of 0.05%. Second, the exponential decay expected assuming the ions to act independently fitted the histogram of decay times t_m obtained from the 2- and 3-ion data well. Third, the decay rates from these fits were combined to estimate the lifetime of the shelved state, giving a result consistent with our previous precise measurement performed on a single ion [17]. Fourth, rank correlation tests were performed on the decay times obtained from the 2- and 3-ion data; no evidence for rank correlation was found.

We suggest therefore that the correlations which have been reported are likely to be due not to interactions between the ions themselves, but to external time-dependent perturbations. In our own trap, we have investigated and reduced such perturbations to a negligible level [17], and the present work demonstrates that when this is done there is no evidence that an ion trap is subject to unexplained effects which would make it unsuitable for quantum information processing.

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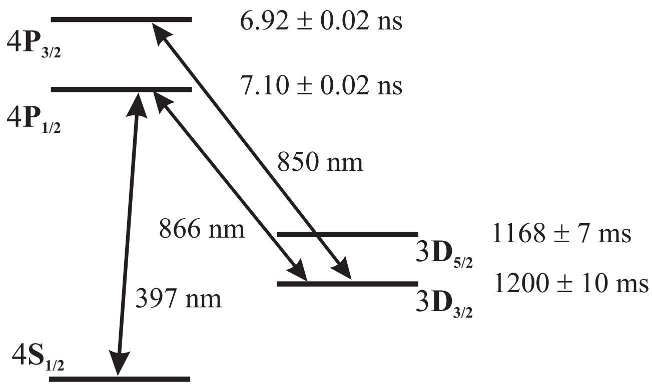

FIG. 1. Low-lying energy levels of ${}^{40}Ca^{+}$, with their lifetimes. Lasers at 397 nm, 866 nm and 850 nm drive the corresponding transitions in the experiments.

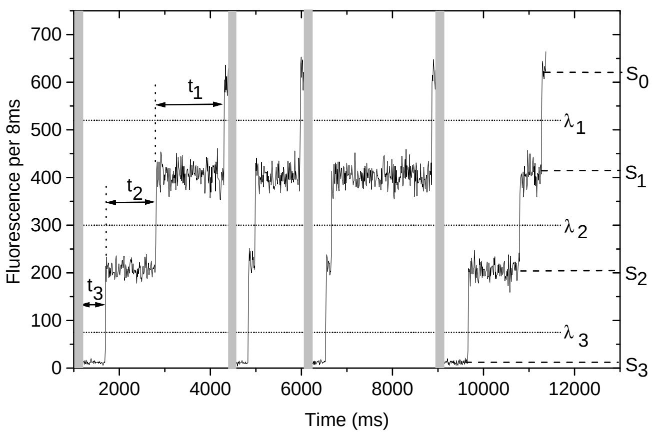

FIG. 2. Observed fluorescence signals from a linear 3-ion crystal. The vertical axis is the number of counts given by the photomultiplier during one 10 ms counting bin (2 ms dead time). The grey bars indicate re-shelving periods, when the shutter on the 850 nm laser was open. The de-shelving times, t_m, are labelled for one observation of the 3 ions decaying from the shelved state, where m is the number of ions remaining to decay. The dotted horizontal lines show the threshold settings λ_m for the data analysis; the dashed horizontal lines show the mean count levels S_m.

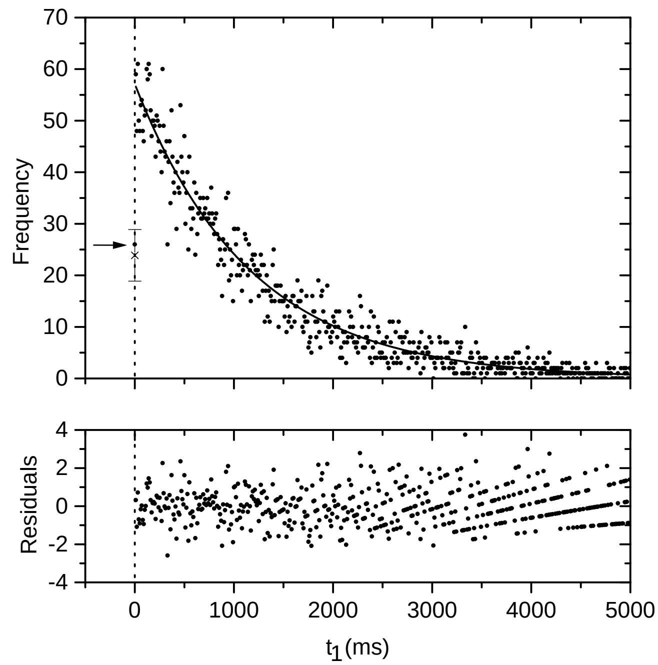

FIG. 3. The histogram of the decay times, t_1, of the last ion of 2 to decay, obtained from a 3.2 hour run, with an exponential $A \exp(-\gamma_1 t)$ fitted to all bins but the first two. In this case, the analysis gave $A = 57 \pm 1$, $\gamma_1 = 0.860 \pm 0.012$ s^{-1}, which agrees with the expected rate $\gamma_1 = 1/\tau = 0.856 \pm 0.005$ s^{-1}, where τ is the lifetime derived from single-ion data [17]. The residuals are shown on an expanded scale, in the form (data−fit)/$\sqrt{\text{fit}}$. The first bin gives the number of 2-ion jumps observed to be coincident within one counting bin and has a modified bin width (see text), which reduces the expected number in the first bin to be $F = 0.42$ of the value, $f_1 = 57$, predicted by the fitted exponential. The expected number, $Ff_1 = 24 \pm 5$ (marked with a cross), agrees with the observed number, 26 (indicated by an arrow).

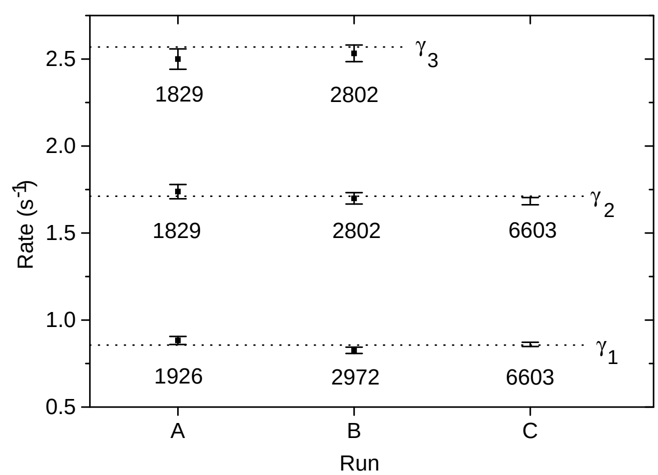

FIG. 4. Measured de-shelving rates γ_m of the the next ion to decay from the state where m ions are shelved; errors are purely statistical. The horizontal lines are the expected rates $\gamma_m = m/\tau$ if the ions are acting independently, where τ is the lifetime derived from single-ion data [17] and have negligible error on this scale. Runs A and B were conducted with 3 ions, run C with 2 ions. The number below each point gives the number of decay times in the corresponding histogram.

Run	N	Time (hrs)	$m_i \to m_f$	N_{QJ}	n_{c}	n_{obs}
A	3	1.1	$2 \to 0$	1926	7.0	10
			$3 \to 1$	1829	9.5	9
			$\mathbf{3 \to 0}$	**1829**	**0.02**	**0**
B	3	1.7	$2 \to 0$	2972	10.5	13
			$3 \to 1$	2802	15.4	13
			$\mathbf{3 \to 0}$	**2802**	**0.03**	**0**
C	2	3.2	$2 \to 0$	6603	23.9	26
total 2-fold		6.0	$(2,3) \to (0,1)$	16132	66.3	71
total 3-fold		**2.8**	$\mathbf{3 \to 0}$	**4631**	**0.05**	**0**

TABLE I. Two-fold and three-fold (bold type) coincident quantum jumps, with N ions. Coincident quantum jumps occur with m_i ions initially shelved, leaving m_f ions shelved. N_{QJ} is the total number of quantum jumps observed with m_i ions initially shelved. For independent ions, n_{c} of these jumps are predicted to be coincident, taking into account the modified bin width. n_{obs} gives the number of coincidences observed. The third column gives the total amount of time that one or more ions spent shelved in each run.

Run	N	R_{12}	R_{23}	R_{13}	$R^{95\%}$
A	3	-0.025	-0.010	-0.018	0.046
B	3	-0.019	$+0.010$	$+0.008$	0.037
C	2	$+0.008$	—	—	0.024

TABLE II. Results of the rank correlation tests, with N ions. R_{nm} is the Spearman rank-order correlation coefficient for the decay times t_n and t_m. $|R_{nm}|$ would have to be greater than $R^{95\%}$ for 95% significance [19].